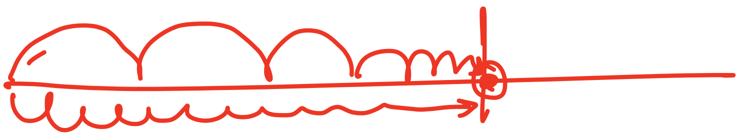
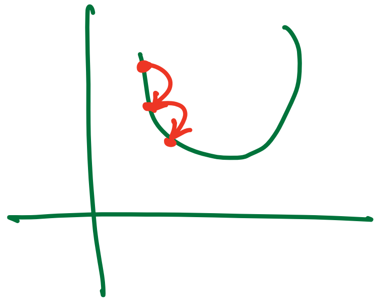
$\eta \rightarrow$ learning rate adaptive





A travel agency wants an automated system to predict travel costs. The agency has the following data available with it.

Table II

S. No.	Distance (in Km) (x)	Travelling Cost (in Rupees) (y)
x^1 1	1	2.75
x^2 2	2	3.5
x^3 3	3	4.25
x^4 4	4	5
x^5 5	5	5.75

$\rightarrow$ Regression

[redacted]

Formulate the above problem as a linear model $h(x) = w_0 + w_1 x$ to predict the travelling cost for a given distance. The parameter w_0 is 2 (optimal). Apply gradient descent algorithm to find optimal parameter w_1. The learning rate for the first epoch is 0.073, and for the second epoch and later, the learning rate is 0.091. Let the initial value of w_1 is 0.5.

$$\theta_1 := \theta_1 - \eta \frac{\partial J(\theta)}{\partial \theta_1} \rightarrow \theta_1 := \theta_1 - \frac{\eta}{m} \cdot 2 \sum_{i=1}^{m} \left(\hat{y}^{(i)} - y^{(i)}\right) x^{(i)}$$

initialize $\rightarrow 0.5$

do

$J(\theta)$

update $\theta \rightarrow \eta = 0.073$

$\eta = 0.091$

while(converge)

1st epoch

$x^{(i)}$	$h_\theta(x^{(i)})$ $\hat{y}^{(i)}$	$y^{(i)}$	$\hat{y}^{(i)} - y^{(i)}$	$(\hat{y}^{(i)} - y^{(i)}) x^{(i)}$
1	$2+0.5*1 = 2.5$	2.75	-0.25	-0.25
2	$2+0.5*2 = 3$	3.5	-0.5	-1

3	$2+0.5*3 = 3.5$	4.25	-0.75	-2.25
4	$2+0.5*4 = 4$	5	-1	-4
5	$2+0.5*5 = 4.5$	5.75	-1.25	-6.25

$\Sigma = -13.75$

$\Sigma (h_\theta(x^{(i)}) - y^{(i)}) x^{(i)}$

$$h_\theta(x^{(i)}) = \theta_1 x^{(i)} + \theta_0$$
$$= 0.5\, x^{(i)} + 2$$

$$\theta_1 = \theta_1 - \frac{\eta * 2}{m} \Sigma (h_\theta(x^{(i)}) - y^{(i)}) x^{(i)}$$

$$\theta_1 = 0.5 - \frac{0.073 * 2}{5} (-13.75)$$

$$\theta_1 = 0.9$$

2nd epoch

repeat same process with $\theta_1 = 0.9$

Linear Regression with Multiple features

Eg: House Price Prediction

features (x) (Prediction)

	#Area	#floors	#Bedrooms	#Age	Price (y)
	x_1	x_2	x_3	x_4	
$x^{(1)}$	200	2	3	10	2.5
$x^{(2)}$	100	3	2	20	2
⋮	⋮				

$$X = \begin{bmatrix} \text{———} & x^1 & \text{———} \\ \text{———} & x^2 & \text{———} \\ & \vdots & \\ \text{———} & x^m & \text{———} \end{bmatrix}$$

m examples

$$X = \begin{bmatrix} x^1_1 & x^1_2 & x^1_3 & \cdots & x^1_n \\ x^2_1 & x^2_2 & x^2_3 & & x^2_n \\ \vdots & & & & \\ x^m_1 & x^m_2 & x^m_3 & \cdots & x^m_n \end{bmatrix}$$

$$\underbrace{m}_{\text{examples}} \times \underbrace{n}_{\text{features}}$$

x^i_j = ith example jth feature

Hypothesis

$\hat{y} = h_\theta(x) = \theta_0 + \underbrace{\theta_1 x}_{\text{single feature}}$

(lurs) x
0
0
0
0

x_1	x_2	x_3	$\cdots$	x_n

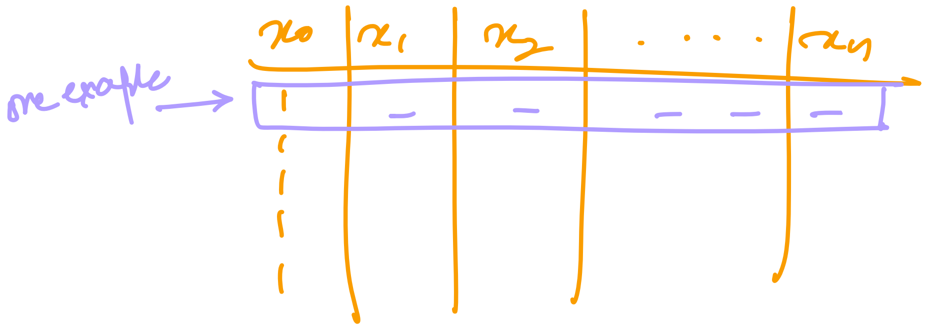
$$\hat{y} = h_\theta(x) = \underbrace{\theta_0}_{\text{bias}} + \underbrace{\theta_1 x_1 + \theta_2 x_2 + \cdots \theta_n x_n}_{\text{weight assign feature}}$$

$$h_\theta(x) = \theta_0 + \sum_{i=1}^{n} \theta_i x_i$$

$$h_\theta(x) = \theta_0 x_0 + \sum_{i=1}^{n} \theta_i x_i \qquad x_0 = 1$$

$$h_\theta(x) = \sum_{i=0}^{n} \theta_i x_i$$

$n+1$ features



$$h_\theta(x) = \sum_{i=0}^{n} \theta_i x_i = \theta_0 x_0 + \theta_1 x_1 + \cdots \theta_n x_n$$

$$\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix} \qquad x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$\theta^T = \begin{bmatrix} \theta_0 & \theta_1 & \theta_2 & \cdots & \theta_n \end{bmatrix} \qquad x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$\theta^T x = \theta_0 x_0 + \theta_1 x_1 + \cdots \theta_n x_n$$

$$h_\theta(x) = \sum_{i=0}^{n} \theta_i x_i = \theta^T x$$

- Random
- Good your θ is ? $\rightarrow$ error
- Update your θ $\rightarrow$ G.D

Error/loss fxn:
$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m}\left(\hat{y}^{(i)} - y^{(i)}\right)^2$$

Update θ:
$$\theta = \theta - \eta \frac{\partial J(\theta)}{\partial \theta}$$

Gradient

$\nabla_\theta J(\theta)$

$$\theta = [\theta_0\ \theta_1\ \theta_2\ \ldots\ \theta_n]$$

$$\frac{\partial J(\theta)}{\partial \theta_j} = \frac{\partial}{\partial \theta_j}\left(\frac{1}{m}\sum_{i=1}^{m}\left(\hat{y}^{(i)} - y^{(i)}\right)^2\right)$$

$$= \frac{1}{m}\sum_{i=1}^{m}\frac{\partial}{\partial \theta_j}\left(\underline{\hat{y}^{(i)}} - y^{(i)}\right)^2$$

$$= \frac{1}{m}\sum_{i=1}^{m}\frac{\partial}{\partial \theta_j}\left(\underline{\theta_0 x_0 + \theta_1 x_1 + \ldots \theta_j x_j + \ldots \theta_n x_n} - y^{(i)}\right)^2$$